Path-Planning For Formation Flying Applications

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Collision Avoidance Guidance for Formation Flying
Formation Flying Spacecraft & The Need for Collision Avoidance

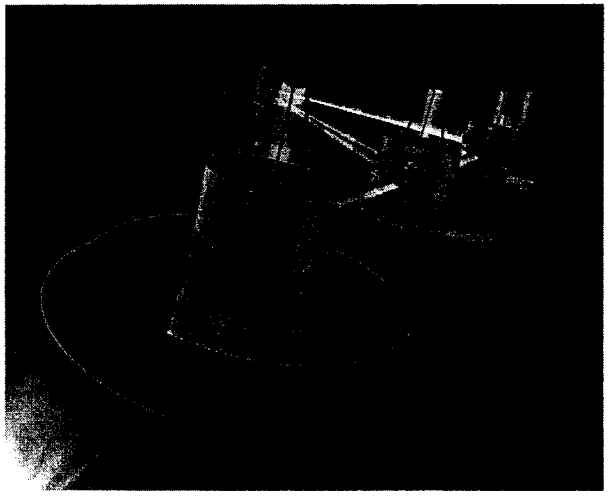
Formation flying spacecraft refers to a set of spatially distributed spacecraft flying in formation, capable of interacting and cooperating with one another in a coordinated fashion

- Enables distributed sensing, data collection, co-observations, and global communications for earth applications
- Enables long variable baseline space interferometery for deep space applications, e.g. TPF – the Terrestrial Planet Finder, TPI – the Terrestrial Planet Imager, Starlight, LISA

Formation flying spacecraft must undergo frequent formation reconfigurations, during which

- the spacecraft-to-spacecraft separations can range from a few meters to several kilometers during reconfigurations
- it is mission critical to avoid collisions between spacecraft as they move in space

Earth Obserations

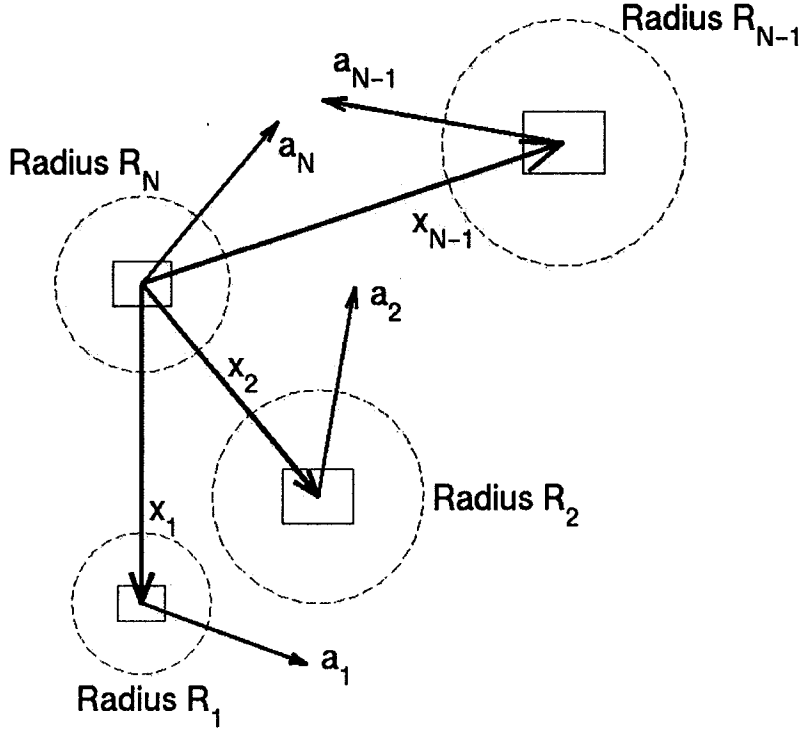


Terrestrial Planet Finder

Collision Avoidance Guidance for Formation Flying
Notation

Consider an N - spacecraft formation, for which we define:

- x_k Linear position of spacecraft k with respect to the N^{th} spacecraft in inertial coordinates, $k = 1, 2, .., N\text{-}1$
- v_k Relative velocity; $v_k = \dot{x}_k$, $k = 1, 2, .., N\text{-}1$
- a_k Absolute linear acceleration of the k^{th} spacecraft, $k = 1, 2, .., N$
- R_k The radius of the exclusion sphere about k^{th} spacecraft , $k = 1, 2, .., N$
- T Reconfiguration maneuver time



Collision Avoidance Guidance for Formation Flying Assumptions

The formation reconfiguration maneuvers are of the rest-to-rest variety

i.e. $v_k(t) = 0,\ k = 1,\ 2,\ ..,\ N\text{-}1,$ at $t = 0,\ t = T$. Almost all reconfigurations for interferometry applications belong to this class. The one which does not is the formation synchronized rotations as a single monolithic unit. Avoiding collisions is not a concern in this case since the specific maneuver places additional constraints on the motion, which preclude collisions.

The natural orbital perturbations on system relative equations of motion are small enough to be ignored

This is a realistic assumption for the deep-space formation flying application under consideration here. Orbital dynamics induced relative motion accelerations are several orders of magnitude below the path accelerations during formation reconfigurations. The assumption renders the equations of motion linear.

Spacecraft rotational degrees of freedom are ignored

This is not restrictive from an application standpoint. This requires that either a prescribed fraction of the total acceleration capability be used for translation path-planning (the balance reserved for attitude planning) or a momentum exchange device be used for attitude control.

Collision Avoidance Guidance for Formation Flying
The Formation Collision Avoidance Problem

Evaluating appropriate path accelerations $a_k(t),\ k = 1, 2, .., N,$ **which minimize**

$$J = \frac{1}{2} \int_0^T \left\{ \sum_{k=1}^{N} (a_k \bullet a_k) \right\} dt$$

Subject to:

$$\dot{x}_k = v_k, \qquad k = 1, 2, .., N\text{-}1,$$

$$\dot{v}_k = a_k - a_N, \qquad k = 1, 2, .., N\text{-}1,$$

$$x_k(0) = x_{k0}, \qquad x_k(T) = x_{kT}, \qquad k = 1, 2, .., N\text{-}1,$$

$$v_k(0) = 0, \qquad v_k(T) = 0, \qquad k = 1, 2, .., N\text{-}1,$$

$$\| x_k(t) - x_j(t) \| \geq (R_k + R_j), \qquad k, j = 1, 2, .., N\text{-}1;\ k \neq j, \qquad t \in [0, T],$$

$$\| x_k(t) \| \geq (R_k + R_N), \qquad k = 1, 2, .., N\text{-}1; \qquad t \in [0, T],$$

$$| a_{k_i}(t) | \leq A_{k_i}, \quad i = x, y, z; \qquad k = 1, 2, .., N; \qquad t \in [0, T].$$

Collision Avoidance Guidance for Formation Flying
Possible Solution Approaches

Move the member spacecraft one at a time during reconfigurations:

Reduces complexity

Very sub-optimal from the standpoint of any metric

Ground-planned reconfigurations:

Compute collision-free paths on the ground, and uplink using a suitable parameterization

Non-real time, labor intensive, becomes difficult to manage as N increases

Published works deal with similar problems (Robotics, space rendezvous applications) using artificial potential function methods, but there are problems with these approaches:

Constraints are not satisfied exactly

Prone to traps and limit-cycle behavior

Our contribution:

Problem formulated as a parameter optimization problem whose size is proportional to N , the number of spacecraft

An iterative algorithm to solve the problem on-board

Sub-optimal solutions which are suitable for real-time implementation

Collision Avoidance Guidance for Formation Flying
Change of Variables

Define: $\xi = t/T$, $(\cdot)' = d(\cdot)/d\xi$, which allows the following restatement of the problem

Minimize:

$$J = \frac{1}{2} T \int_0^1 \left\{ \sum_{k=1}^{N} (a_k \bullet a_k) \right\} d\xi,$$

Subject to:

$$x_k' = T\ v_k, \qquad k = 1, 2, .., N\text{-}1,$$

$$v_k' = T\ (a_k - a_N), \qquad k = 1, 2, .., N\text{-}1,$$

$$x_k(0) = x_{k0}, \quad x_k(1) = x_{kT}, \qquad k = 1, 2, .., N\text{-}1,$$

$$v_k(0) = 0, \quad v_k(1) = 0, \qquad k = 1, 2, .., N\text{-}1,$$

$$\| x_k(\xi) - x_j(\xi) \| \geq (R_k + R_j), \qquad k, j = 1, 2, .., N\text{-}1;\ k \neq j, \qquad \xi \in [0, 1],$$

$$\| x_k(\xi) \| \geq (R_k + R_N), \qquad k = 1, 2, .., N\text{-}1; \qquad \xi \in [0, 1],$$

$$| a_{ki}(\xi) | \leq A_{ki}, \quad i = x, y, z; \qquad k = 1, 2, .., N; \qquad \xi \in [0, 1].$$

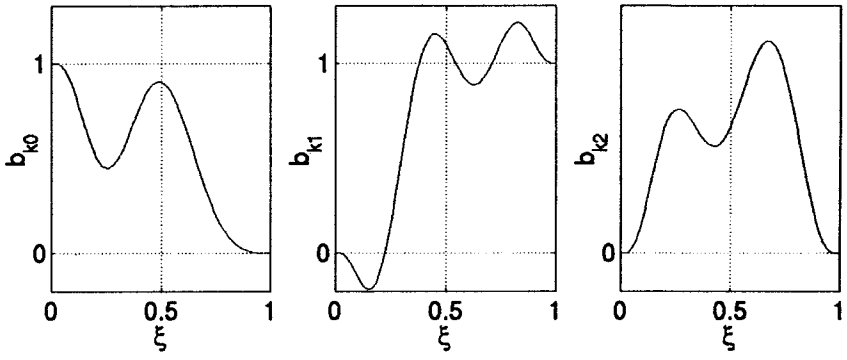
Collision Avoidance Guidance for Formation Flying
Solution Approach: Path Parameterization

Express the optimal solution trajectories as:

$$x_k^*(\xi) = b_{k0}(\xi)\, x_{k0} + b_{k1}(\xi)\, x_{kT} + b_{k2}(\xi)\, x_{k0} \times x_{kT},\ k = 1, 2, \ldots, N\text{-}1, \quad \xi \in [0, 1].$$

Functions $b_{ki}(\xi)$, $i = 0, 1, 2,$ are continuously differentiable functions (Path Functions) of ξ which satisfy:

$$b_{k0}(0) = 1,\ b_{k0}(1) = 0,\ b_{k0}'(0) = 0,\ b_{k0}'(1) = 0,$$
$$b_{k1}(0) = 0,\ b_{k1}(1) = 1,\ b_{k1}'(0) = 0,\ b_{k1}'(1) = 0,$$
$$b_{k2}(0) = 0,\ b_{k2}(0) = 1,\ b_{k2}'(0) = 0,\ b_{k2}'(1) = 0.$$



Observations about the chosen parameterization:

1. Defines a feasible path.
2. Equivalent to the most general representation possible.
3. Trivial to place additional constraints on optimal paths, e.g. it may be required to further restrict optimal paths such that they lie in the plane spanned by the end points (set $b_{k2}(\xi) \equiv 0$). This is the case for the TPF mission, for example, where reconfiguration maneuvering in close proximity must be constrained in this fashion to ensure thermal protection.
4. The problem now: determine the optimal set $\{b_{ki}(\xi)\}$, $\xi \in [0, 1]$.

Collision Avoidance Guidance for Formation Flying
Path Functions

We make the following choice:

$$b_{ki}(\xi) = \sum_{j=0}^{n} c_{kij}\, \xi^j, \quad \xi \in [0, 1].$$

Observation:

The optimal solution of the constraint-free optimal path-planning problem belongs to this class of solutions where $c_{kij} = 0,\ k = 1, 2, .., N\text{-}1;\ i = 0, 1, 2;\ j \geq 4$

$$\Rightarrow x_k(\xi)^* = (1 - 3\xi^2 + 2\xi^3)\, x_{k0} + (3\xi^2 - 2\xi^3)\, x_{kT},\ k = 1, 2, .., N\text{-}1.$$

Enforce boundary conditions:

$$b_{k0}(\xi) = 1 - 3\xi^2 + 2\xi^3 + \sum_{j=4}^{n} [\,(j\text{-}3)\,\xi^2 - (j\text{-}2)\,\xi^3 + \xi^j\,]\, c_{k0j}, \qquad k = 1, 2, .., N\text{-}1,$$

$$b_{k1}(\xi) = 3\xi^2 - 2\xi^3 + \sum_{j=4}^{n} [\,(j\text{-}3)\,\xi^2 - (j\text{-}2)\,\xi^3 + \xi^j\,]\, c_{k1j}, \qquad k = 1, 2, .., N\text{-}1,$$

$$b_{k2}(\xi) = \sum_{j=4}^{n} [\,(j\text{-}3)\,\xi^2 - (j\text{-}2)\,\xi^3 + \xi^j\,]\, c_{k2j}, \qquad k = 1, 2, .., N\text{-}1.$$

$\Rightarrow 3\,(N\text{-}1)\,(n\text{-}3)$ undetermined constants (c_{kij}'s); over-parameterized system requires $n > 3 + N/6$

Collision Avoidance Guidance for Formation Flying
Numerical Algorithm

Ignore all acceleration constraints

Treat time T as another parameter, determined *a posteriori* to satisfy acceleration constraints; trivial, since accelerations vary as $\sim 1/T^2$

Problem: Determine set $C \triangleq \{c_{kij}\}$ which minimizes J/T subject to only the collision avoidance constraints

Non-convex optimization (convex cost, non-convex constraints)

Define minimum spacecraft-to-spacecraft separations and gradients

$$d_{kj} = \underset{\xi \in [0,1]}{Minimum} \; || \, x_k(\xi) - x_j(\xi) \, ||, \qquad k, j = 1, 2, .., N\text{-}1, \quad j \neq k,$$

$$d_{kN} = \underset{\xi \in [0,1]}{Minimum} \; || \, x_k(\xi) \, ||, \qquad k = 1, 2, .., N\text{-}1.$$

$$\nabla J = (1/T) \, \partial J / \partial C,$$

$$\nabla d_{kj} = \partial d_{kj} / \partial C, \qquad k = 1, 2, .., N\text{-}1; \; j = 1, 2, .., N; \quad j \neq k.$$

Simple gradients-based search for an optimal (local) C

Analytic integration of equations of motions $\Rightarrow d_{kj}$

Numerical evaluations of the gradients $\Rightarrow$ form weighted $\nabla d_{kj} = \nabla d$

Search direction σ satisfies: $\sigma \bullet \nabla J \leq 0, \;\; \sigma \bullet \nabla d > 0 \Rightarrow$ Update C along σ

Collision Avoidance Guidance for Formation Flying
Evaluation of Maneuver Duration

Necessary and sufficient conditions for optimality $\Rightarrow \sum_{k=1}^{N} a_k(\xi) = 0,\ \xi \in [0, 1]$, which implies:

$$a_k(t) = \frac{1}{T^2}\left[\frac{N-1}{N}\, x_k''(\xi) - \frac{1}{N}\sum_{\substack{j=1 \\ j \neq k}}^{N-1} x_j''(\xi)\right], \quad k = 1, 2, .., N-1,$$

$$a_N(t) = \frac{1}{T^2}\left[-\frac{1}{N}\sum_{k=1}^{N-1} x_k''(\xi)\right].$$

Numerical solution to the problem also yields evaluations for

$$\alpha_{ki} = \underset{\xi \in [0, 1]}{Maximum}\left[\frac{N-1}{N}\, x_{ki}''(\xi) - \frac{1}{N}\sum_{\substack{j=1 \\ j \neq k}}^{N-1} x_{ji}''(\xi)\right], \quad i = x, y, z;\ k = 1, 2, .., N-1,$$

$$\alpha_{Ni} = \underset{\xi \in [0, 1]}{Maximum}\left[-\frac{1}{N}\sum_{k=1}^{N-1} x_{ki}''(\xi)\right], \quad i = x, y, z.$$

$$\Rightarrow T = \sqrt{\underset{\substack{k \in [1,2,..,N] \\ i \in [x,y,z]}}{Maximum}\left[\frac{\alpha_{ki}}{A_{ki}}\right]}$$

Collision Avoidance Guidance for Formation Flying
Numerical Examples $(N = 2)$

x_{10} = (10.0, 2.0, 0.0) m,

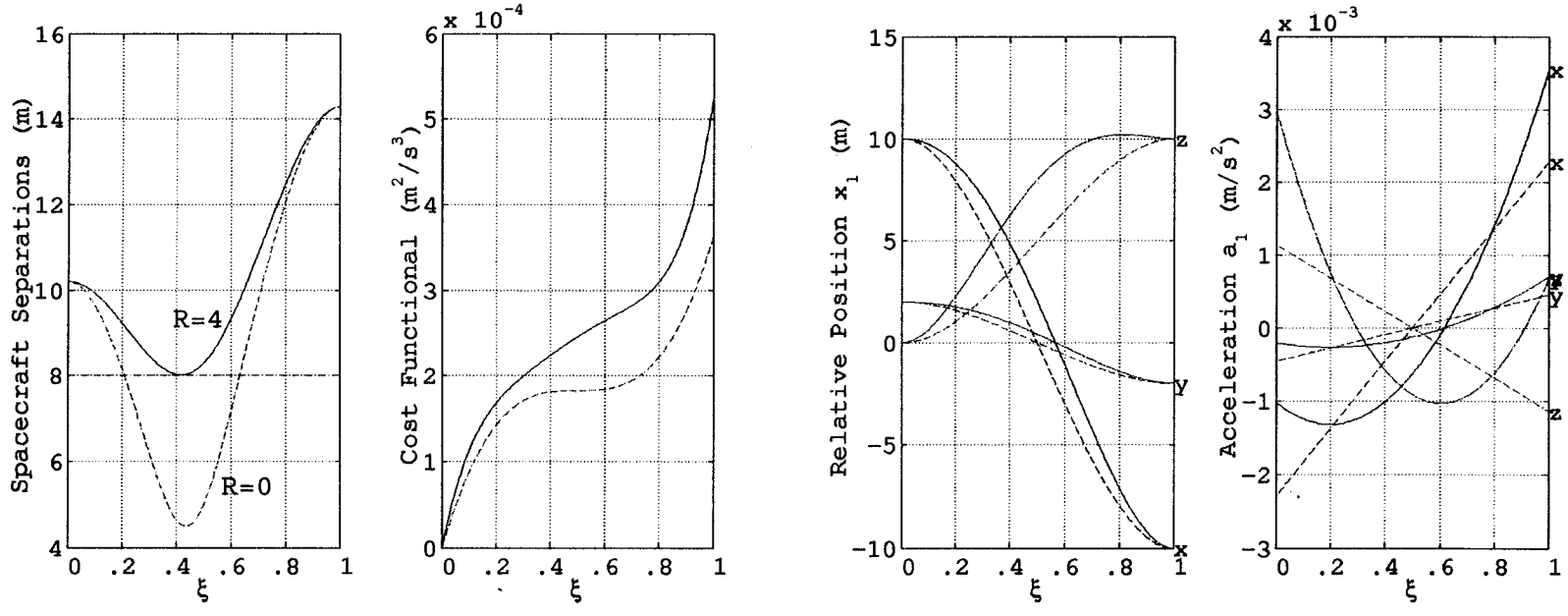
x_{1T} = (-10.0, -2.0, 10.0) m,

A_1 = (0.005, 0.004, 0.003) m/s^2,

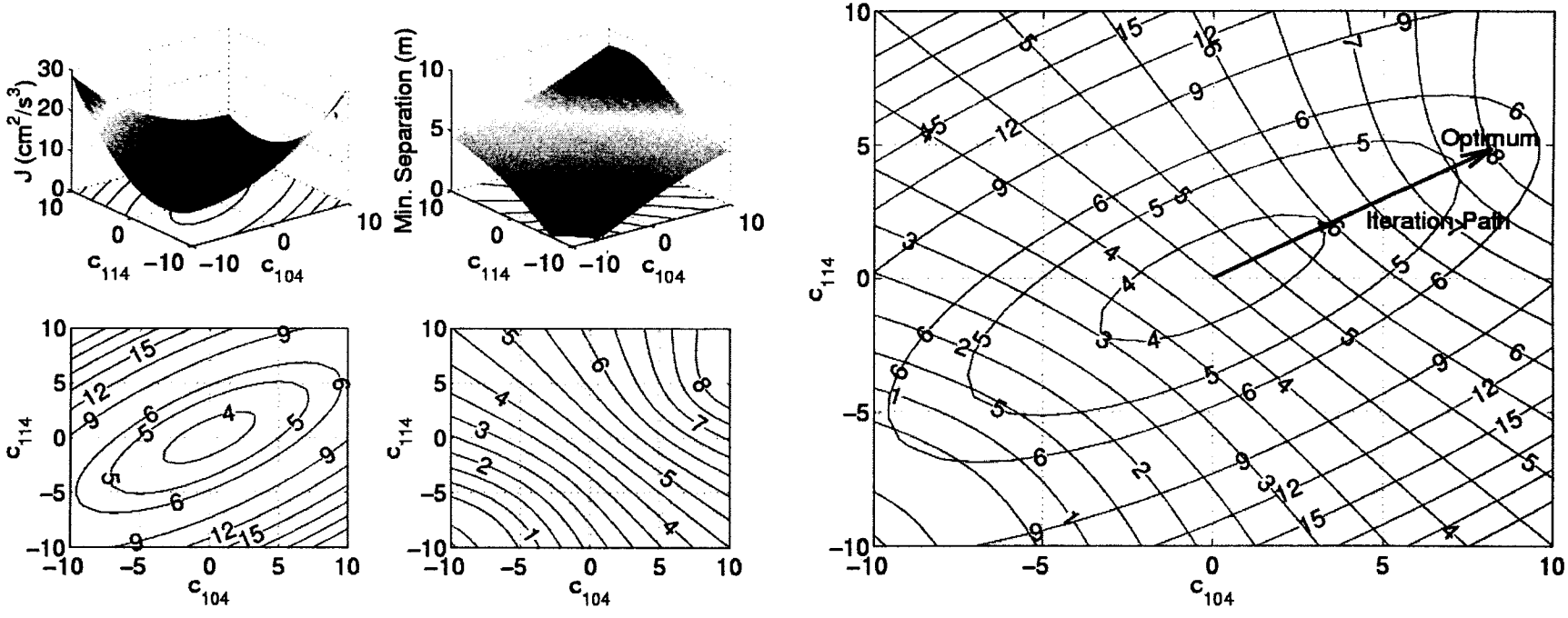
A_2 = (0.004, 0.003, 0.005) m/s^2,

$R_1 = R_2 = 4.0$ m.

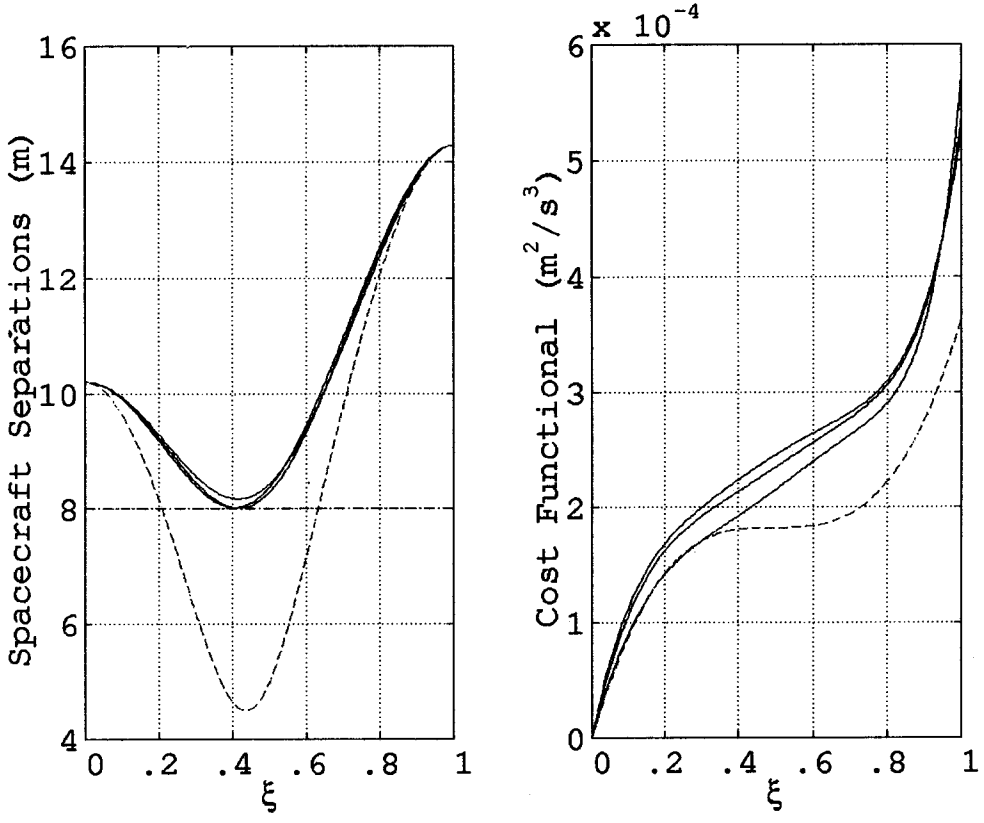
$n = 4 \Rightarrow C = (c_{104}, c_{114}, c_{124}) = (8.148, 4.838, -8.4e\text{-}6)$, $T = 162$ sec.



Collision Avoidance Guidance for Formation Flying
Numerical Examples $(N = 2,\ n = 4)$



Collision Avoidance Guidance for Formation Flying
Numerical Examples $(N = 2,\ n = 5,\ 6)$

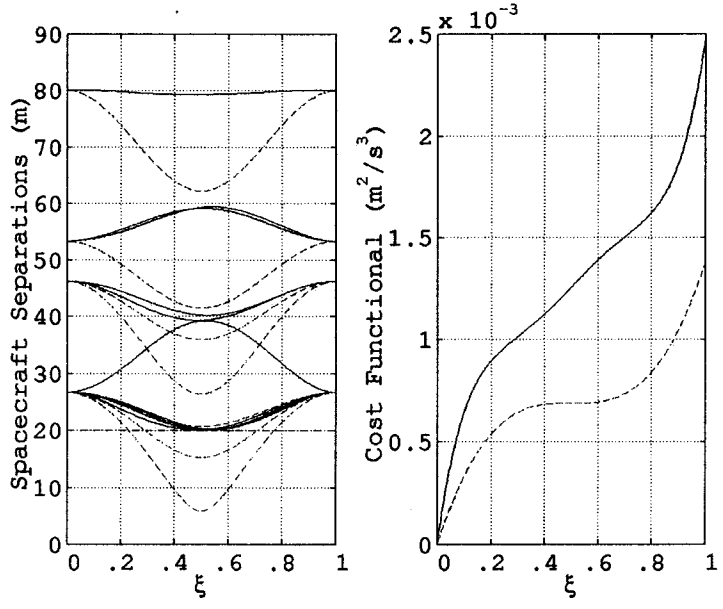


Collision Avoidance Guidance for Formation Flying
Numerical Examples *(TPF: $N = 5$)*

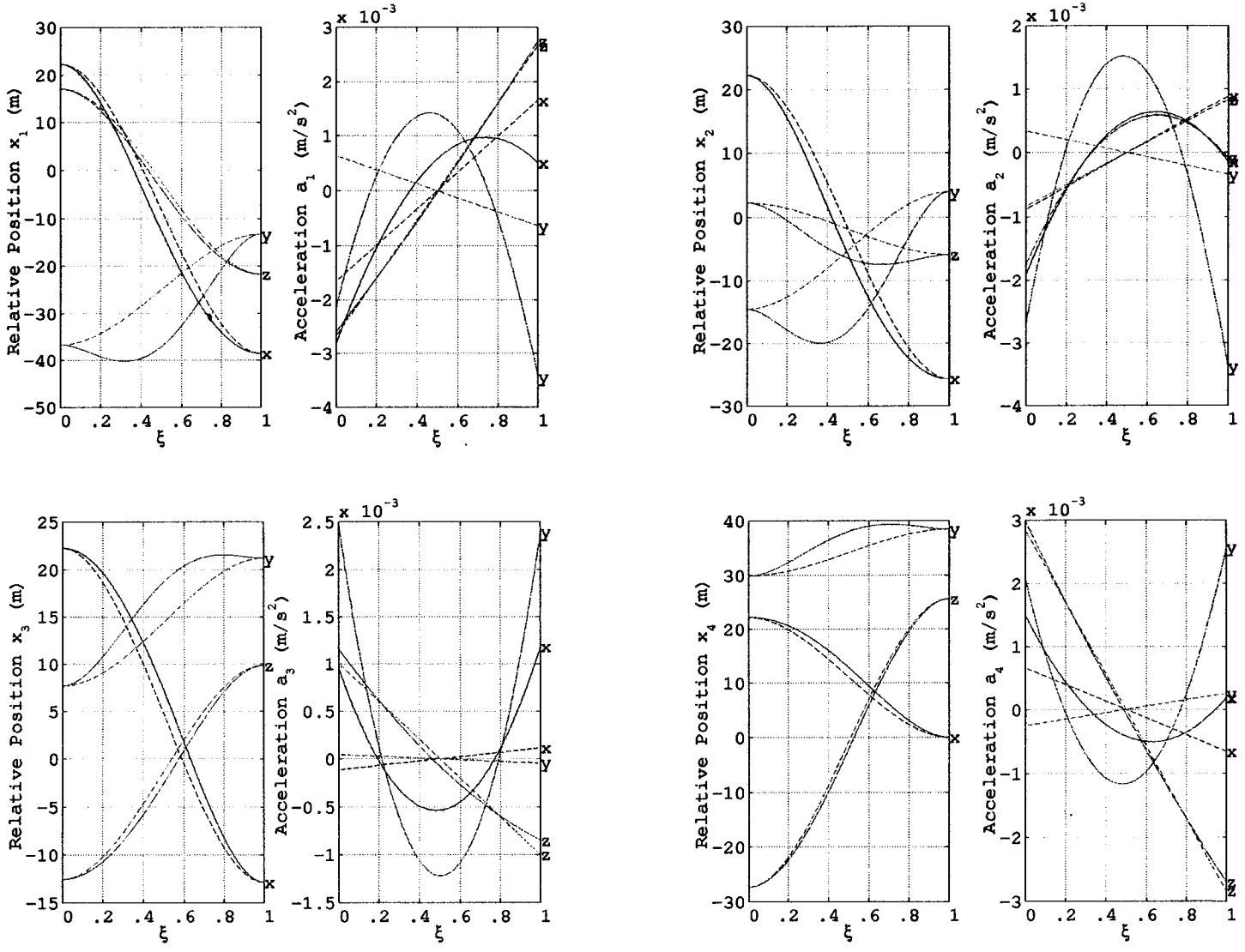
x_{10} = (22.254, -36.706, 17.052) m, x_{1T} = (-38.545, -13.285, -21.705) m,
x_{20} = (22.254, -14.518, 2.2605) m, x_{2T} = (-25.697, 3.9533, -5.9300) m,
x_{30} = (22.254, 7.6703, -12.532) m, x_{3T} = (-12.848, 21.192, 9.8453) m,
x_{40} = (22.254, 29.8580, -27.324) m, x_{4T} = (0.000, 38.431, 25.621) m,

A_k = (0.005, 0.004, 0.003) m/s^2, $(k \neq 5)$; A_5 = (0.004, 0.003, 0.005) m/s^2, R_k = 10.0 m, $k = 1, 2, .., 5$.

$n = 4 \Rightarrow$ $(c_{104}, c_{114}, c_{124})$ = (3.7976, 4.0480, 0.6680); $(c_{204}, c_{214}, c_{224})$ = (19.448, 19.394, 2.6337),
$(c_{304}, c_{314}, c_{324})$ = (3.0534, 2.0985, -2.3686); $(c_{404}, c_{414}, c_{424})$ = (1.3019, 0.6078, -0.4503),
T = 316 sec.



Collision Avoidance Guidance for Formation Flying
Numerical Examples *(TPF: $N = 5$)*



Collision Avoidance Guidance for Formation Flying Conclusions

The problem of minimum energy, acceleration-constrained collision avoidance problem for formation flying applications is considered and a solution in presented

Minimum energy closely related to fuel expenditure

Other costs may also be considered in the proposed framework

Sub-optimal, but easy to implement algorithm

The proposal seeks sub-optimal solutions which are attractive from the standpoint of real-time implementations. The solution is sub-optimal since it tries to locally minimize the appropriate cost-functional within the class of paths under consideration.

Guaranteed convergence only in the case of a 2-spacecraft formation

No failures to converge yet

May be possible to offer some guarantees for the general case

It appears that, within the class of proposed solutions, consideration of only the first significant term in the time-series approximation yields a solution with the lowest cost

Additional work needed in the following areas

Inclusion of inequality constraints on relative velocities

Avoidance assured path planning which guarantees collision-free motions in the case of a "failure" mid-stream